Isoscaling behavior in the Fission Dynamics

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The fission processes of ^{112}Sn + ^{112}Sn and ^{116}Sn + ^{116}Sn are simulated with the combination of the Langevin equation and the statistical decay model. The masses of two fission fragments are given by assuming the process of symmetric fission or asymmetric fission by the Monte Carlo sampling with the Gaussian probability distribution. From the analysis to the isotopic/isotonic ratios of the fission fragments from both reactions, the isoscaling behavior has been observed and investigated in details. Isoscaling parameters α and β are extracted as a function of the charge number and neutron number, respectively, in different width of the sampling Gaussian probability distribution. It seems that α is sensitive to the width of fission probability distribution of the mass asymmetrical parameter but β is not. Both α and β drop with the increasing of beam energy and the reduced friction parameter.

I. INTRODUCTION

The availability of exotic nuclear beam with extreme neutron-to-proton ratio provides an opportunity to explore the collision dynamics of isospin-asymmetric nuclear systems [1]. To facilitate this kind of study, the suitable selection of the sensitive experimental observables related to the isospin degree of freedom is one of key points. One of such observables is the isotopic/isobaric ratio [2, 3], which has been used to probe the isospin equilibration at medium energies before. Recently, this kind of ratios have been systematically revisited for two different reactions with the same charge number and the similar temperature and a so-called isoscaling law has been observed experimentally [4, 5, 6]. Isoscaling means that the ratio of isotope yields from two different reactions, 1 and 2, $R_{21}(N, Z) = Y_2(N, Z)/Y_1(N, Z)$, is found to exhibit an exponential relationship as a function of the neutron number N and proton number Z [4]

$$R_{21}(N, Z) = \frac{Y_2(N, Z)}{Y_1(N, Z)} = C exp(\alpha N + \beta Z), \quad (1)$$

where C, α and β are three parameters. In grand-canonical limit, $\alpha = \Delta\mu_n/T$ and $\beta = \Delta\mu_z/T$ where $\Delta\mu_n$ and $\Delta\mu_z$ are the differences between the neutron and proton chemical potentials for two reactions, respectively. This behavior is attributed to the difference of two reaction systems with different isospin asymmetry. It is potential to probe the isospin dependent nuclear equation of state by the studies of isoscaling [7]. So far, the isoscaling behavior has been experimentally explored in various reaction mechanisms, ranging

from the evaporation [4], fission [8, 9] and deep inelastic reaction at low energies to the projectile fragmentation [10, 11] and multi-fragmentation at intermediate energy [4, 12, 13]. While, the isoscaling phenomenon has been extensively examined in different theoretical frameworks, ranging from dynamical transport models, such as Blotzmann-Uehling-Uhlenbeck model [12], Quantum Molecular Dynamics model [14] and Anti-symmetrical Molecular Dynamics model [15], to the statistical models, such as the expansion emission source model, the statistical multi-fragmentation model and the lattice gas model [5, 6, 16, 17, 18]. In this work, we will focus on the detailed simulation studies on the isoscaling behavior of the fission fragments. A brief report has been published recently [19].

In this work, we present an analysis for the fragments from the fission which was simulated by the Langevin equation. The isotopic or isotonic ratios of the different fragment yields from ^{116}Sn + ^{116}Sn and ^{112}Sn + ^{112}Sn system are presented and the features of isoscaling behavior in fission dynamics are investigated.

The paper is organized by the following structure. In Sec. II, a brief description of the Langevin model is given and the partition of masses of two fission fragments is assumed; In Sec. III, the detailed results for the fission-fragment isotopic and isotonic distribution are presented and the isoscaling behavior is explored; Finally we summarize the present work.

II. BRIEF DESCRIPTION OF THE LANGEVIN MODEL

The process of fission can be described in terms of collective motion using the transport theory [20, 21, 22, 23, 24]. The dynamics of the collective degrees of freedom is typically described using the Langevin or Fokker-Planck

equation. In this work, we deal with a Combined Dynamical and Statistical Model (CDSM) which is a combination of a dynamical Langevin equation and a statistical model to describe the fission process of heavy ion reaction [21]. This model is an overdamped Langevin equation coupled with a Monte Carlo procedure allowing for the discrete emission of light particles. It switches over to statistical model when the dynamical description reaches a quasi-stationary regime. We first specify the entrance channel through which a compound nucleus is formed,

i.e. the target and projectile is complete fusion. The fusion process of simulating the fission in each trajectory with angular momentum $L = \hbar l$ is described by

$$\sigma(l) = \frac{2\pi}{k^2} \frac{2l+1}{1 + exp[(l-l_c)/\delta_l]} \tag{2}$$

where the parameters l_c and δl is according to an approximating scaling of Ref. [20]. Namely,

$$l_c = \sqrt{A_P \times A_T/A_{\mathrm{CN}}} \times (A_P^{1/3} + A_T^{1/3}) \times (0.33 + 0.205 \times \sqrt{E_{\mathrm{c.m.}} - V_c}) \tag{3}$$

when $0 < E_{\mathrm{c.m.}} - V_c < 120$ MeV; and when $E_{\mathrm{c.m.}} - V_c > 120$ MeV the term in the last bracket is put equal to 2.5. In the above equation, A_T and A_P represents the mass of target and projectile, respectively, and A_{CN} is the mass of compound nucleus. For the barrier V_c the following ansatz is used:

$$V_c = \frac{5}{3} c_3 \times \frac{Z_P Z_T}{A_P^{1/3} + A_T^{1/3} + 1.6}, \tag{4}$$

with $c_3 = 0.7053$ MeV. The diffuseness δl is found to scale as

$$\delta l = \{ \begin{matrix} (A_P A_T)^{3/2} \times 10^{-5} \times [1.5 + 0.02 \times (E_{\mathrm{c.m.}} - V_c - 10)] & \text{for } E_{\mathrm{c.m.}} > V_c + 10 \\ (A_P A_T)^{3/2} \times 10^{-5} \times [1.5 - 0.04 \times (E_{\mathrm{c.m.}} - V_c - 10)] & \text{for } E_{\mathrm{c.m.}} < V_c + 10 \end{matrix}$$

Trajectory with the particular angular momentum L is started at the ground state position q_{gs} of the entropy $S(q_{\mathrm{gs}}, E_{\mathrm{tot}}^*, A, Z, L)$, q is half of the distance between the centers of masses of the future fission fragments. In this work the total initial excitation energy E_{tot}^* is given by $E_{\mathrm{tot}}^* = E_{\mathrm{beam}} A_T/(A_T + A_P) + Q$ where Q is the fusion Q-value calculated by $Q = M_T + M_P - M_{\mathrm{CN}}^{\mathrm{LD}}$. M_T and M_P is the mass of projectile and target come from experimental data, respectively. If it is unavailable, it is calculated by macroscopic-microscopic model [25]. $M_{\mathrm{CN}}^{\mathrm{LD}}$ is the mass of the compound nucleus which is calculated from the liquid-drop model.

The dynamical part of CDSM model is described by the Langevin equation which is driven by the free energy F. F is related to the level density parameter $a(q)$ [26]

$$F(q, T) = V(q) - a(q)T^2 \tag{5}$$

in the Fermi gas model, where $V(q)$ is the fission potential and T is the nuclear temperature.

The overdamped Langevin equation reads

$$\frac{dq}{dt} = -\frac{1}{M\beta_0(q)} \left(\frac{\partial F(q,T)_T}{\partial q} \right) + \sqrt{D(q)}\Gamma(t), \tag{6}$$

where q is the dimensionless fission coordinate defined as above. $\beta_0(q)$ is the reduced friction parameter which is the only parameter of this model. The fluctuation strength coefficient $D(q)$ can be expressed according to the fluctuation-dissipation theorem:

$$D(q) = \frac{T}{M\beta_0(q)}, \tag{7}$$

where M is the inertia parameter which drops out of the overdamped equation. $\Gamma(t)$ is a time-dependent stochastic variable with Gaussian distribution. Its average and correlation function is written as

$$< \Gamma(t) >= 0,$$
$$< \Gamma(t)\Gamma(t') >= 2\delta_\varepsilon(t - t'). \tag{8}$$

The potential energy $V(A, Z, L, q)$ is obtained from the finite-range liquid drop model [27]

$$V(A, Z, L, q) = a_2[1 - k(\frac{N-Z}{A})^2]A^{2/3}[B_s(q) - 1]$$
$$+ c_3 \frac{Z^2}{A^{1/3}}[B_c(q) - 1] + c_r L^2 A^{-5/3} B_r(q), \tag{9}$$

where $B_s(q)$, $B_c(q)$ and $B_r(q)$ means surface, Coulomb and rotational energy terms, respectively, which depends on the deformation coordinate q. a_2, c_3, k and c_r are parameters not related to q. In our calculation we take them according to Ref. [20].

$$a_2 = 17.9439\text{MeV}, \quad c_3 = 0.7053\text{MeV},$$
$$k = 1.7826, \qquad c_r = 34.50\text{MeV}.$$

We use c and h [28] to describe the shape of nucleus,

$$\rho^2(z) = (1 - \frac{z^2}{c_0^2})((\frac{1}{c^3} - \frac{b_0}{5})c_0^2 + B_{\text{sh}}(c,h)z^2), \quad (10)$$

where

$$c_0 = cR, \quad R = 1.16A^{1/3}. \quad (11)$$

The nuclear shape function $B_{\text{sh}}(c,h)$ and the collective fission coordinate $q(c,h)$ of mass number A is expressed as

$$B_{\text{sh}}(c,h) = 2h + \frac{c-1}{2},$$
$$q(c,h) = \frac{3}{8}c(1 + \frac{2}{15}B_{\text{sh}}(c,h)c^3). \quad (12)$$

The fission process of the Langevin equation is propagated using an interpretation of Smoluchowski [32]($\lambda = 1$ in the following equation) which is consistent with the kinetic form which reads

$$q_{n+1} = q_n + [\frac{T(q)}{\beta_0(q)M}\frac{dS(q)}{dq}]_n\tau + \lambda[\frac{d}{dq}(\frac{T(q)}{\beta_0(q)M})]_n\tau + \sqrt{[\frac{T(q)}{\beta_0(q)M}]_n\tau}w_n \quad (13)$$

Here τ is the time step of the Langevin equation, w_n is a Gaussian distributed random number with variance 2. $S(q) = 2\sqrt{a(q,A)[E_{\text{tot}} - V(q,A,Z,l)]}$ is the entropy. The parameter λ allows us to distinguish between the different possibilities to discretize the Langevin equation. It is called interpretation in the literature. In the analysis of the experiments on fission of hot nuclei discussed in the review [20, 29] and in the papers quoted there, the Itô-interpretation ($\lambda = 0$) [30] has been used exclusively. Also there are other interpretations, namely that of Stratonovich [31] ($\lambda = 1/2$), or an interpretation which is consistent with the kinetic form of the Smoluchowski equation of [32] ($\lambda = 1$). In this work, we take $\lambda = 1$.

In our calculation we adopt one-body dissipation (OBD) friction form factor β_{OBD} [33] as $\beta_0(q)$ which is calculated with one-body dissipation with a reduction of wall term except the special case which we claim. Here we use an analytical fit formula which was developed in Ref. [34], i.e.

$$\beta_{\text{OBD}}(q) = \{ \begin{array}{ll} 15/q^{0.43} + 1 - 10.5q^{0.9} + q^2 & \text{if } q > 0.38 \\ 32 - 32.21q & \text{if } q < 0.38 \end{array}$$

In the dynamical part of the model the emission of light particles (n, p, d, α) and giant dipole γ are calculated at each Langevin time step τ, the widths for particle and giant dipole γ decay are given by the parametrization of Blann [35] and Lynn [36], respectively.

III. RESULTS AND DISCUSSIONS

A. Isotopic/Isotonic distributions of the fission fragments

In Fig. 1 (a) and (b), we demonstrate that the ratio of the pre-scission neutron number R_N and of the pre-scission proton number R_P between ^{116}Sn + ^{116}Sn and ^{112}Sn + ^{112}Sn, respectively, as a function of beam energy (E_{beam}/A). First, the values of R_N is larger than 1 while those of R_P is less than 1, indicating that the neutron is easier to be emitted for neutron-rich system while the proton is in contrary trend. Of course, this is a natural result from the chemical composition of reaction system [37, 38]. Second, with the increasing beam energy, R_N shows a decreasing trend while R_P in reverse way, which can be interpreted the isospin effect weakens as the beam energy rises up.

Within the framework of the Langevin simulation we chose 200,000 fission events which happen on dynamic channel (we give up the events which happen in statistical part of CDSM model) and chose a Gaussian distributed random number as the mass asymmetry parameter $\alpha_0 = \frac{A_1 - A_2}{A_1 + A_2}$ when the system reaches to the scission point. When $\alpha_0 = 0$ it means the symmetrical fission. It is taken from a Gaussian distributed random number from -1 to 1 with the mean value of zero. A_1 and A_2 is the mass of the two fission fragments, respectively. In this work we assume the fission fragments have the same N/Z as the one of the initial system and then Z_1 or Z_2 of fission fragments can be deduced from A_1 or A_2. This
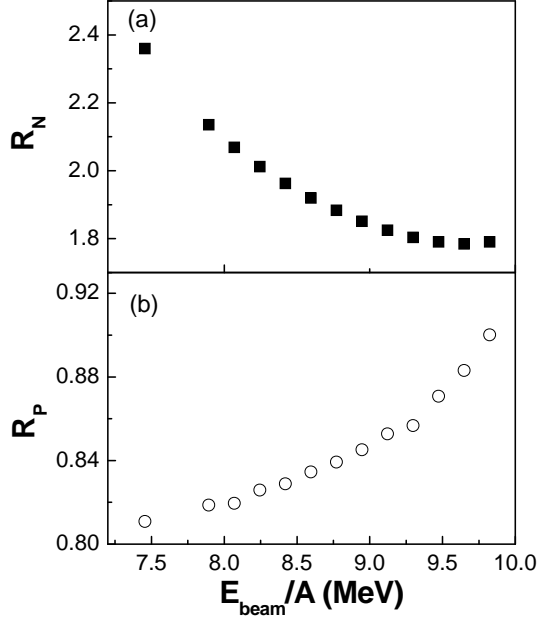
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FIG. 1: The ratios of the pre-scission neutron number (a) and of the pre-scission proton number between ^{116}Sn + ^{116}Sn and ^{112}Sn + ^{112}Sn as a function of beam energy (E_{beam}/A).

FIG. 2: (Color online) The fission-fragment mass distributions produced by the Langevin simulation for the reactions of ^{112}Sn + ^{112}Sn (open symbols) and ^{116}Sn + ^{116}Sn (filled symbols) at 8.4 MeV/nucleon with the different sampling width (σ_{α_0}) of Gaussian probability distribution.

assumption is similar to the case of deep inelastic heavy ion collisions at low energies, where the isospin degree of freedom has been found to reach equilibrium first [39]. Fig. 2 shows the mass distribution of the fission fragments from ^{112}Sn + ^{112}Sn and ^{116}Sn + ^{116}Sn reaction systems assuming the different width of the sampling Gaussian probability for the mass asymmetrical parameter of fission fragments (σ_{α_0}). Naturally, the bigger the σ_{α_0}, the wider the fragment mass distributions.

Samples for the isotopic and isotonic distributions in some given Z and N are shown in Fig. 3. The square of the full widths of these distributions shows a systematic increase with the Z or N as shown in Fig. 4 and the absolute value of the differences of the centroid of the isotonic/isotopic distributions shows an increasing trend too (see Fig. 6). Apparently, the widths are not sensitive to the width of the Gaussian probability for the mass asymmetrical parameter of fission fragments, but the differences of the centroid of the isotonic/isotopic distributions shows the dependence on it.

From a practical point of view, the isoscaling behavior occurs when two mass distributions for a given Z from two processes with different isospin are Gaussian distributions with the same width but different mean mass. In this case, the isotopic distribution in a given Z, namely $Y(N)|_Z$, and isotonic distribution in a given N, namely $Y(Z)|_N$, can be described by single Gaussian distribu-
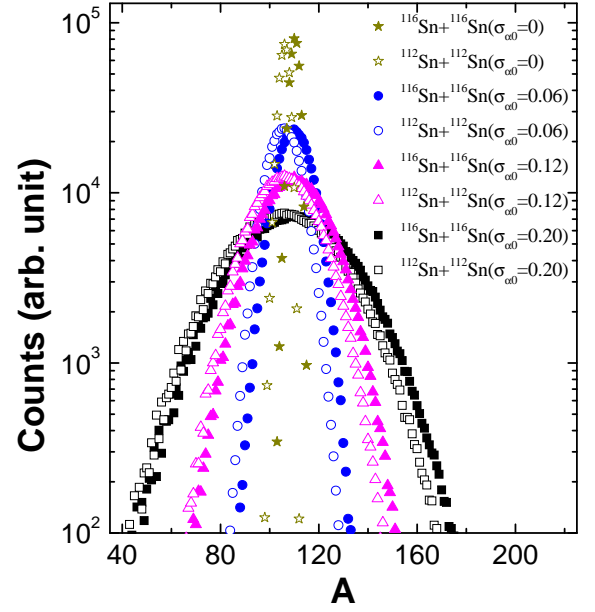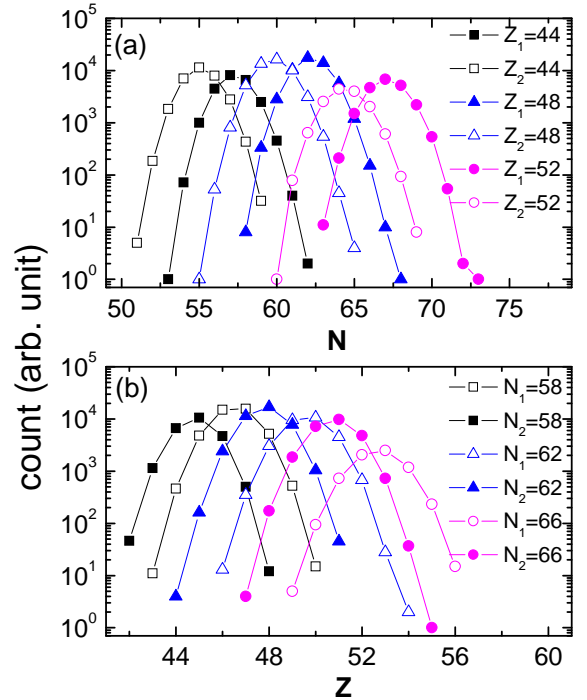

FIG. 3: (Color online) The isotopic (a) and isotonic (b) distributions of fission-fragments in some given Z or N (see texts in the inserts) from the collisions of ^{116}Sn + ^{116}Sn (filled symbols) and of ^{112}Sn + ^{112}Sn (open symbols) at 8.4 MeV/nucleon. $\sigma_{\alpha_0} = 0.06$. Notice that the scale of X-axis is different.

tion, respectively, i.e.:

$$Y(N)|_Z \sim exp[-\frac{(N-N_Z)^2}{2\sigma_Z^2}],$$

$$Y(Z)|_N \sim exp[-\frac{(Z-N_N)^2}{2\sigma_N^2}], \qquad (14)$$

where N_Z and N_N are the centroid of isotopic and isotonic distributions, σ_Z^2 and σ_N^2 describe the variance of distributions for each element of charge Z and neutron number N, respectively. This leads to an exponential behavior of the ratio R_{21} if the quadratic term in N_Z or N_N is neglected, it reads

$$ln(R_{21}(N)|_Z) \sim \frac{[(N_Z)_2 - (N_Z)_1]N}{\sigma_Z^2},$$

$$ln(R_{21}(Z)|_N) \sim \frac{[(N_N)_2 - (N_N)_1]Z}{\sigma_N^2}. \qquad (15)$$

Note that Eq.(15) requires the values for σ_Z^2 or σ_N^2 to be approximately the same for both reactions, which is a necessary condition for isoscaling. Indeed, we observed this case in our simulations for both Sn + Sn collisions. In the Langevin equation, σ_Z^2 or σ_N^2 essentially depends on the physical conditions reached, such as the temperature, the density and the friction parameter etc. Considering that $R_{21}(N)|_Z \sim exp(\alpha N)$ or $R_{21}(Z)|_N \sim exp(\beta Z)$ for a given Z or N, we can get

$$\alpha \sim \frac{(N_Z)_2 - (N_Z)_1}{\sigma_Z^2},$$

$$\beta \sim \frac{(N_N)_2 - (N_N)_1}{\sigma_N^2}. \qquad (16)$$

Assuming other ingredients can be neglected, σ_Z^2 or σ_N^2 could be considered to be proportional to temperature T of the fission-fragments according to the fluctuation-dissipation theorem [23], in this circumstance,

$$\alpha \sim \frac{(N_Z)_2 - (N_Z)_1}{T},$$

$$\beta \sim \frac{(N_N)_2 - (N_N)_1}{T}, \qquad (17)$$

where $[(N_Z)_2 - (N_Z)_1]$ or $[(N_N)_2 - (N_N)_1]$ can be understood as a term of the average difference of the neutron or proton chemical potential between two reactions.

As we already showed that both σ_Z^2 and σ_N^2 rise with Z and N, respectively, and are almost independent of σ_{α_0} from Fig. 4(a) and (b) in our calculation. On the other hand, we recognize that the similar behavior of the Z-dependence of σ_Z has been experimentally observed in the spallation-fission data of ^{208}Pb (1 GeV/nucleon) + d or p etc in Gesellschaft für Schwerionenforschung (GSI) [40, 41]. According to the model which is based on the modern version of Abrasion-ablation model involving the fission nuclei by Benlliure *et al.* [42], the square of
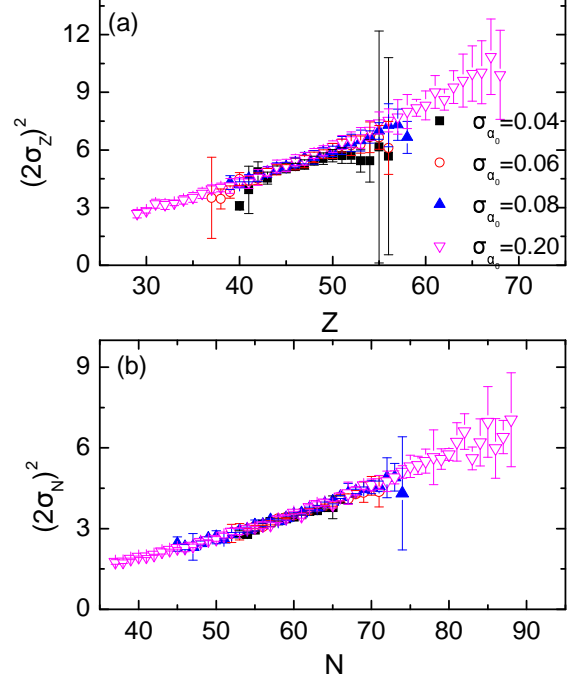

FIG. 4: (Color online) The square of full width of the isotopic (a) and isotonic distribution (b) as a function of Z and N, respectively, in different σ_{α_0}. The width is the average value of the two systems (they are almost the same for both systems).

the width of symmetric fission fragment from the macroscopic potential can be expressed by

$$\sigma_Z^2 = \frac{1}{2}\frac{\sqrt{E_{bf}^*}}{\sqrt{a}C_{mac}} = \frac{T_{fis}}{2C_{mac}}, \qquad (18)$$

where E_{bf}^* is the excitation energy above the the fission barrier, a is level energy parameter, T_{fis} is the temperature of fissioning nuclei and C_{mac} is the curvature of macroscopic potential energy V_{mac} as a function of charge asymmetry. In this way, the width of symmetric fission fragment distribution increases with temperature. This is also the case in our present model calculation. In the other word, the temperature of the fission-fragments which mostly originates from the symmetric fission seems to increases with the charge number of fragments. Recently, a systematic study on the experimental data also displays that the variance of the fragment mass distribution increases with the temperature of the compound nucleus and the fission-fragments [43].

To verify the relationship of the temperature and charge number of fragments as stated above, we extract the temperature of the fissioning nuclei in the scission point when the system happens on dynamic channel. Fig. 5(a) and (b) demonstrate that the mean temperature of two systems as a function of Z and N for the fissioning nuclei, respectively. Obviously, the temperature almost increases linearly with the charge number (Z_{fis})
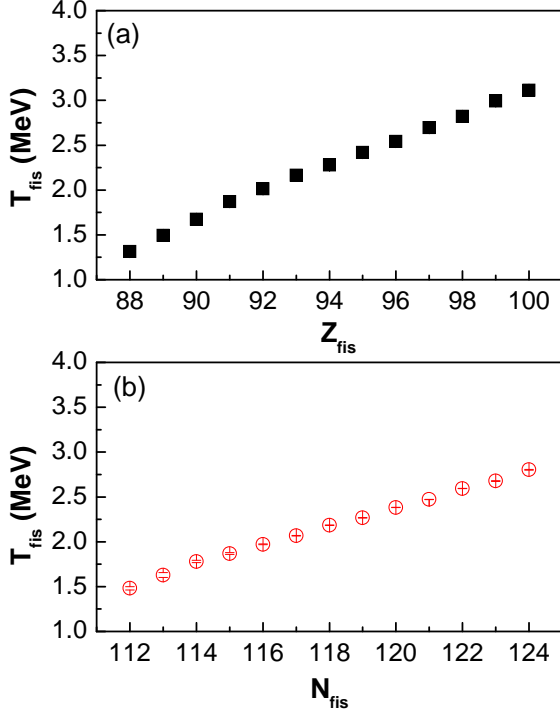
FIG. 5: (Color online) The extracted mean temperature of the fissioning nuclei of two reaction systems as a function of their charge number (top panel) or the neutron number (bottom panel).
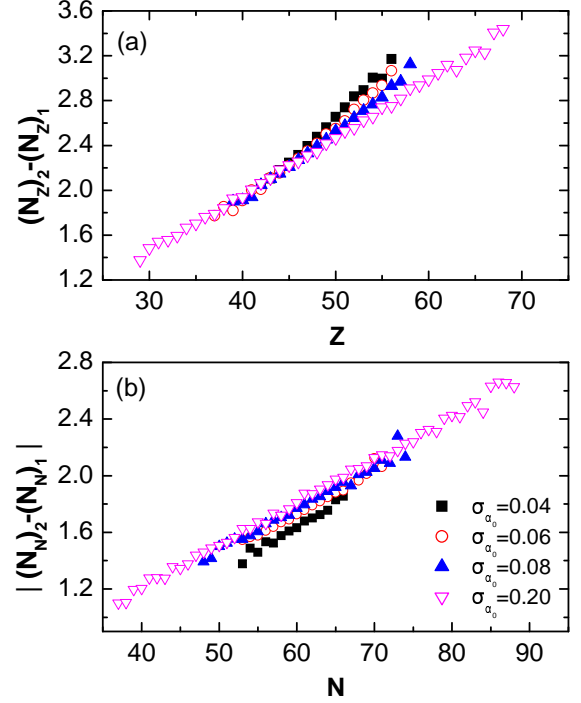

FIG. 6: The absolute value of $(N_Z)_2 - (N_Z)_1$ (a) and $|(N_N)_2 - (N_N)_1|$ (b) as a function of Z and N of the fission-fragments, respectively, between ^{116}Sn + ^{116}Sn and ^{112}Sn + ^{112}Sn with different σ_{α_0}.

or neutron number (N_{fis}) of the fissioning nuclei. Since we assume the fission-fragments have the same N/Z as the one of the fissioning nuclei, hence the temperature of the fission-fragments shall increase with their charge number.

In Eq.(16) $[(N_Z)_2 - (N_Z)_1]$ or $[(N_N)_2 - (N_N)_1]$ can be understood as a term of the average difference of the neutron or proton chemical potential between two reactions. Fig. 6(a) and (b) shows $[(N_Z)_2 - (N_Z)_1]$ and the absolute value of $[(N_N)_2 - (N_N)_1]$ as a function of Z or N, respectively, in different σ_{α_0}. Apparent increasing behavior with Z or N has been observed. In order to understand the increasing behavior of $[(N_Z)_2 - (N_Z)_1]$ or $|[(N_N)_2 - (N_N)_1]|$ as a function of the charge number and the neutron number of fission-fragments, we investigate the fissioning nuclei. For an example, Fig. 7(a) shows the neutron number versus the charge number of the fissioning nucleus for both reaction systems just before the fission takes place. The lines represent the second order polynomial fits to guide the eyes. From the above points, we can extract the $[(N_Z)_2 - (N_Z)_1]|_{\text{fis}}$ as a function of Z_{fis} as shown in Fig. 7(b). Since the fission-fragments are assumed to have the same N/Z as one of the fissioning nuclei, the fission-fragments shall show the similar increasing behavior as the charge number rises. In comparison to the insensitivity of σ_Z or σ_N to σ_{α_0}, $[(N_Z)_2 - (N_Z)_1]$ and $[(N_N)_2 - (N_N)_1]$ shows somehow stronger dependence

on σ_{α_0}, i.e. $[(N_Z)_2 - (N_Z)_1]$ shows larger values in the proton-rich side while $|[(N_N)_2 - (N_N)_1]|$ shows smaller values in the neutron-deficit side as σ_{α_0} becomes smaller in Fig. 6. This essentially originates from the overlap of the different mass partition of two fission-fragments according to the sampling of Gaussian probability distribution for the mass asymmetry when the fission takes place. The symmetric fissions result in the strongest dependence of $[(N_Z)_2 - (N_Z)_1]$ or $|[(N_N)_2 - (N_N)_1]|$ on the charge number or neutron number.

B. Isoscaling behavior

Eq.(1) can be written as $\ln R_{21} = C_Z + \alpha N$, where $C_Z = \ln C + \beta Z$, if we plot R_{21} as a function of N, on a natural logarithmic plot, the ratio follows along a straight line. In Fig. 8 this isoscaling behavior is observed in the Langevin simulation. Here each kind of symbol with a line represents a chain of isotope. From there, the isoscaling parameter α can be extracted directly. Similarly, the isoscaling parameter β can be extracted from the isotonic ratio as shown in Fig. 9 by $\ln R_{21} = C_N + \beta Z$, where $C_N = \ln C + \alpha N$.

From Fig. 8 and Fig. 9, the relationship between α ($|\beta|$) and the charge number Z (N) of the fission fragments can be deduced. In order to investigate the effect
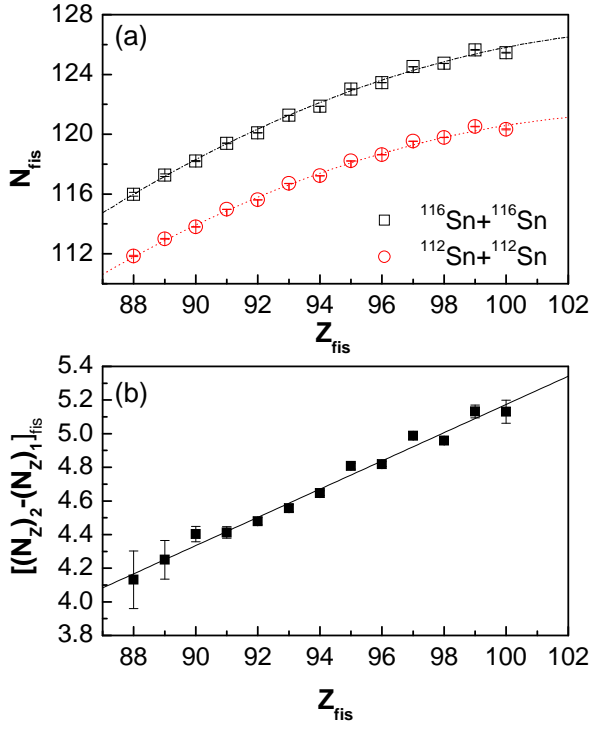
FIG. 7: (Color online) (a) The calculated neutron numbers as a function of the charge numbers for the fissioning nuclei of two reaction systems. (b) The difference of the neutron number of the fissioning nuclei between two reaction systems as a function of the charge number for the fissioning nuclei. The lines represent two-order Polynomial fits in (a) and a linear fit in (b).
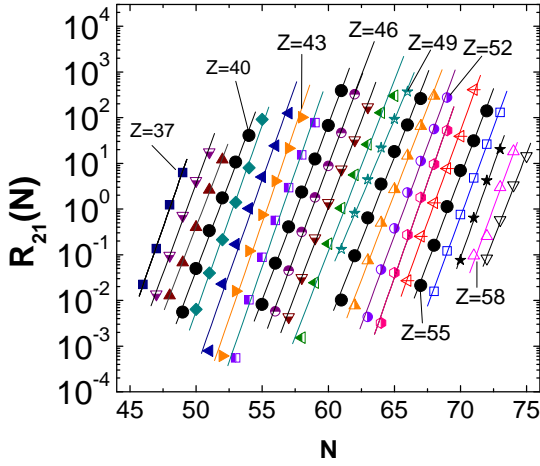
FIG. 9: (Color online) The isotonic yield ratio of the fission fragments between ^{116}Sn + ^{116}Sn and ^{112}Sn + ^{112}Sn in the Langevin simulations at $E_{\rm beam}/A = 8.4$ MeV. Different symbols from left to right represent the calculated results for the isotopes from $N = 46$ - 73. The width of Gaussian probability is assumed 0.06. The lines represent exponential fits to guide the eye.

FIG. 8: (Color online) The isotopic yield ratio of the fission fragments between ^{116}Sn + ^{116}Sn and ^{112}Sn + ^{112}Sn in the Langevin model with $\sigma_{\alpha_0} = 0.06$ and $E_{\rm beam}/A = 8.4$ MeV. Different symbols from left to right represent the calculated results for the isotopes from $Z = 37$ to 59. The lines represent exponential fits to guide the eye.
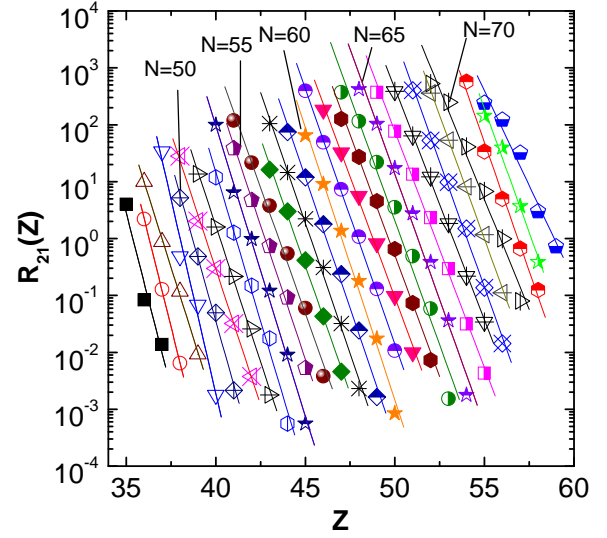
of the width of Gaussian probability distribution on the isoscaling parameters, we use the different widths of the sampling Gaussian distribution for mass asymmetry parameter α_0, namely $\sigma_{\alpha_0} = 0.04$, 0.06, 0.08 and 0.20, with the random number from -1 to 1 and the mean value of 0. Fig.10(a) shows the isoscaling parameter α as a function of Z with different σ_{α_0}. From this figure, we know in the low σ_{α_0}, i.e., when the symmetric fission is an overwhelming mechanism, α increases with Z. This means that the isospin effect becomes stronger with the increasing of Z. In a recent analyse of fission with a simple liquid-drop model [8], a systematic increase of the isoscaling parameter α with the proton number of the fragment element has been predicted. In our simulation, this kind of increase of α with Z apparently stems from the dominated symmetric fission mechanism. While, in the another extreme case from the Fig. 10(a), i.e. with the larger σ_{α_0}, α shows a contrary trend with Z, i.e., it drops with Z. In this case, it seems that there exists stronger isospin effect for the fragments with lower Z. In a medium case, the rising branch and falling branch competes with each other, the mediate isoscaling behavior appears and a minimum of α parameter occurs around the symmetric fission point. We note that the fission data of ^{238,233}U targets induced by 14 MeV neutrons reveal the back-bending behavior of the isoscaling parameter α around the symmetric fission point [9] as stated above. They interpreted that it originates from the temperature difference of fission fragments since the isoscaling parameter is typically, within the grand-canonical approximation,

considered inversely proportional to the temperature ($\alpha = \Delta\mu_n/T$) as stated above. In our case, this kind of backbending of isoscaling parameter α apparently stems from the moderate width of the probability distribution of the mass asymmetrical parameter of the fissioning nucleus as shown in Fig.10. or in the other words, it may stem from a moderate mixture of the different weights between the symmetric and asymmetric fission components. Essentially the backbending originates from the competition between the term of chemical potential and the term of temperature since both terms increase with the charge number of fission fragments.

Besides the above direct method to extract isoscaling parameter, we can also check the behavior of α in terms of Eq. (16). Fig. 10(b) shows the $\frac{[(N_Z)_2-(N_Z)_1]}{\sigma_Z^2}$ as a function of Z. With the increasing of σ_{α_0}, the Z dependence of $\frac{[(N_Z)_2-(N_Z)_1]}{\sigma_Z^2}$ shows from the upswing trend to downswing trend. A turning point around $Z = 51$ is also observed in medium σ_{α_0} as Fig. 10(a) shows. From the similarity of the behavior shown in Fig. 10(a) and (b) as well as the approximate equality of the values of α and $\frac{[(N_Z)_2-(N_Z)_1]}{\sigma_Z^2}$, we can say that the Eq.(16) works well in the present calculation. In our case, the turning point of α stems from the competition between the chemical potential term ($[(N_Z)_2 - (N_Z)_1]$) and the temperature term (σ_Z^2). In general, the chemical potential term is more sensitive to the Gaussian width of the mass asymmetry parameter α_0 for fission fragments (see Fig. 6). Overall speaking, we find that the isoscaling parameter α is sensitive to the width of the probability distribution of mass asymmetrical parameter of the fission fragments. In the other word, we may say that the isoscaling parameter is sensitive to asymmetrical extent of both fission fragments.

Similarly, from Figure. 9, the relationship between $|\beta|$ and the neutron number N of the fission fragments can be deduced in different width σ_{α_0}. This is shown in Fig. 11(a). Different from the relationship of α and Z, the $|\beta|$ always drops with the neutron number, regardless of the change of σ_{α_0}. The quantitative and qualitative similarity of $\frac{[(N_N)_2-(N_N)_1]}{\sigma_N^2}$ vs N (Figure. 11(b)) has also been observed. i.e., it always decreases with N and is insensitive to σ_{α_0}.

However, the obtained isoscaling parameters are actually very large in comparison to the usual isoscaling parameter extracted from the data [9]. The reasons could be the model itself since the model is still too simple as well as our assumption of Gaussian probability distribution of fission fragments. Also the post-fission evaporation component will of course play some roles for modification the isoscaling parameters. In the present model calculation, however, this influence of post-fission evaporation of fission fragment is not included. Those may show larger apparent isoscaling parameters in comparison to the data. Of course, main aim of this work is to show the isoscaling behavior of fission fragments and its trend with the charge or neutron number of the fragments
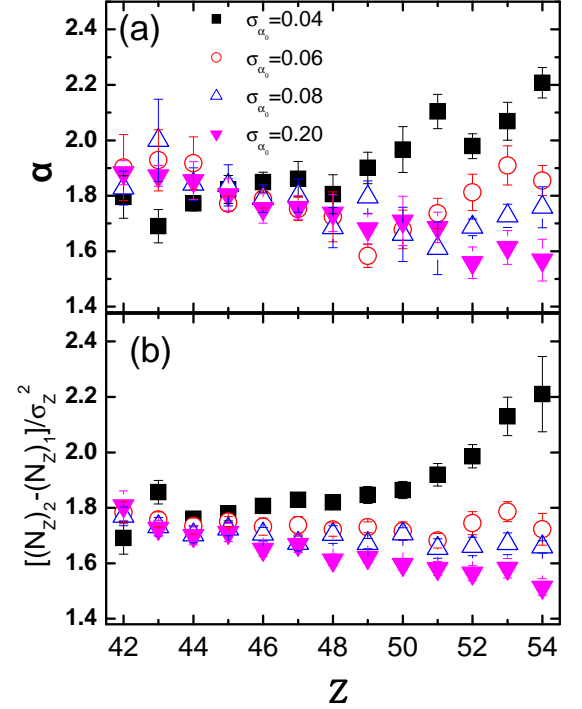

FIG. 10: (Color online) (a) The isoscaling parameter α as a function of Z in the different Gaussian width (σ_{α_0}) of the mass asymmetry parameter α_0 for fission fragments; (b) Same as (a) but for $\frac{[(N_Z)_2-(N_Z)_1]}{\sigma_Z^2}$.

by the Langevin dynamics.

C. The beam energy dependence of the isoscaling parameters

The simulations are systematically performed in different beam energies. The values of α and β are extracted as a function of beam energy for the fragments $Z = 44$ - 54 and $N = 58$ - 68, respectively, as shown in Fig.12(a) and (b). It shows that both α and β decrease as the increasing beam energy which means that the isospin effect fades away with the increasing of E_{beam}/A. This behavior is similar to the case in the fragmentation where the isoscaling parameter drops with the temperature in the statistical models as well as experiments [6, 16, 38, 44].

D. The friction parameter dependence of the isoscaling parameters

In addition, the influence of the reduced friction parameter on the isoscaling parameters is investigated, we use a constant value of $\beta_0 = 2, 4, 6, 8$ and 10 instead of one-body dissipation β_{OBD} which was used in above calculations. In Fig. 13(a) and (b), we plot α and $|\beta|$ as a
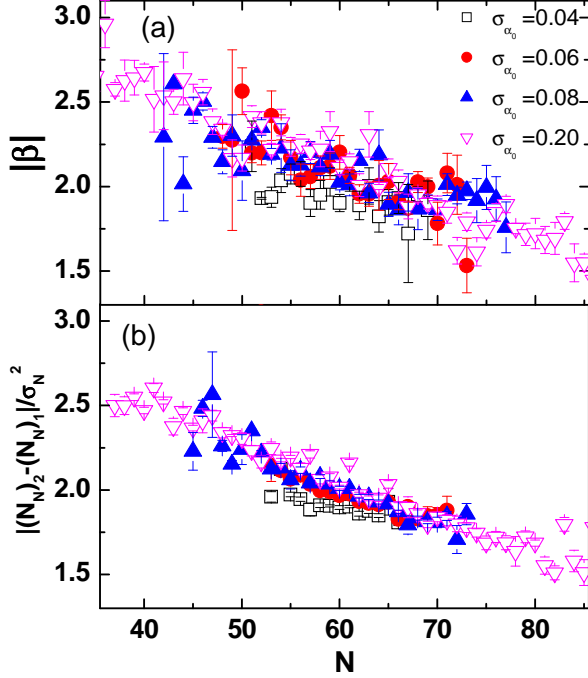
FIG. 11: (Color online) (a) The isoscaling parameter $|\beta|$ as a function of Z in the different Gaussian width (σ_{α_0}) of the mass asymmetry parameter α_0 for fission fragments; (b) Same as (a) but for $|\frac{(N_N)_2-(N_N)_1}{\sigma_N^2}|$.

FIG. 12: (Color online) Isoscaling parameter α (a) and β (b) as a function of beam energy for the fragments $Z = 44 - 54$ and $N = 58 - 68$, respectively. The width (σ_{α_0}) of the Gaussian probability is 0.06.

function of β_0 for different elements from $Z = 44$ to 54 or different isotones from $N = 58$ to 68, respectively. Both α and $|\beta|$ decrease with the increasing of the reduced friction parameter. It shows that α and β are sensitive to the the reduced friction parameter. Larger reduced friction makes the Brownian particles cost more energies which will be transferred to the internal energy from ground state to the scission point than the smaller one, consequently the system will keep less memory to the initial entrance channel. In the viewpoint of the isoscaling behavior, the isoscaling parameter shows a decrease with the reduced friction parameter. Therefore the study on the isoscaling behavior to the fission fragment might be a good tool to explore the friction effect in the fission dynamics process.

IV. SUMMARY

In summary, we applied the Langevin model to investigate the isoscaling behavior in the dynamical process of compound nuclear fission. In order to treat the fission fragments, we assume that the mass asymmetry parameter of the two fission fragments from the fissioning nucleus is taken from a random number with a Gaussian distribution whose width is σ_{α_0}. The simulation illustrates that
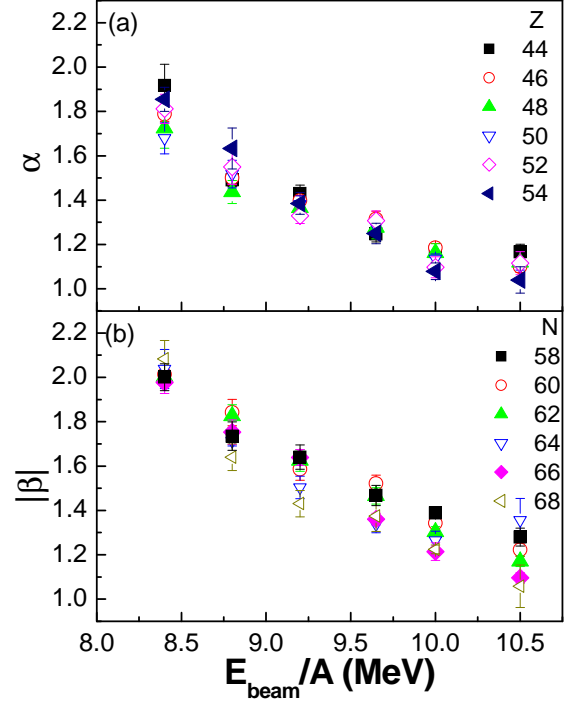
the isotopic and isotonic yield ratios of fission fragments in the dynamical fission channels of ^{116}Sn $+ ^{116}$Sn and ^{112}Sn $+ ^{112}$Sn reaction system show the isoscaling behavior. The terms which are related to the difference of neutron or proton chemical potential are also extracted. It is of interesting that the isoscaling parameter α is sensitive strongly to the Gaussian width σ_{α_0} of the mass asymmetry parameter but β looks not. When σ_{α_0} is small, i.e. the fission is almost symmetric, α increases with the atomic number of fission fragments, which is similar to the theoretical prediction of a simple liquid-drop model [8]. In contrary, when σ_{α_0} is large, for instance, $\sigma_{\alpha_0} = 0.20$, α drops with Z of fission fragments. However, in the intermediate values of σ_{α_0}, α shows a backbending with Z of fission fragments, which is similar to the observation of the ^{238,233}U fission data induced by 14 MeV neutrons [9]. In this context, we could say that the α parameter is sensitive to the asymmetric extent of the fission-fragments from the fissioning nuclei. However, β parameter is insensitive to the width σ_{α_0} even though it always shows the dropping trend with N.

In addition, the dependences of beam energy and the reduced friction parameter for the isoscaling parameters are systematically investigated. It is found that both α and β drop with beam energy of the projectile as well as the reduced friction parameter, reflecting the
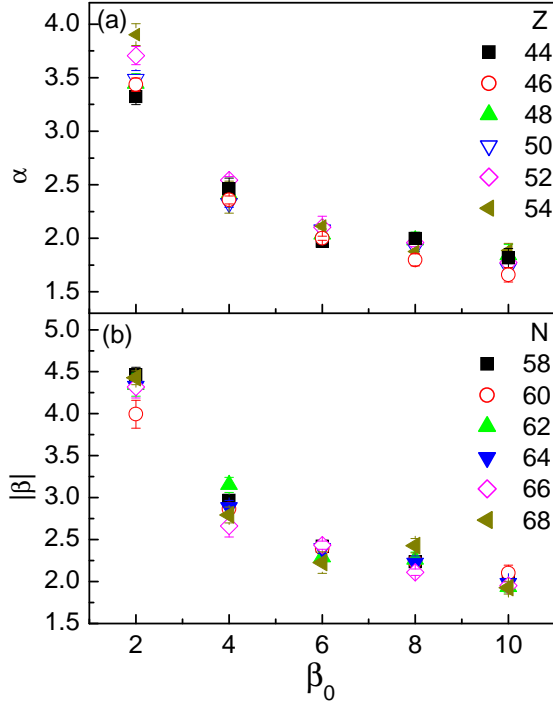
temperature-like dependence of isoscaling parameters in the fission dynamics. The disappearance of isospin effect of fission dynamics is expected in a certain higher beam energy or larger reduced friction parameter. In general, the isoscaling analysis of the fission data appears to be a sensitive tool to investigate the fission dynamics.

FIG. 13: (Color online) α and $|\beta|$ as a function of the reduced friction parameter β_0 for different fission fragments with $Z = 44$ - 54 and $N = 58$ - 68, respectively. The width (σ_{α_0}) of the Gaussian probability is 0.06.

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